Pearson Edexcel

Model Solutions

Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Public release version)

Resource Set 1: Topic 5

Trigonometry

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1.

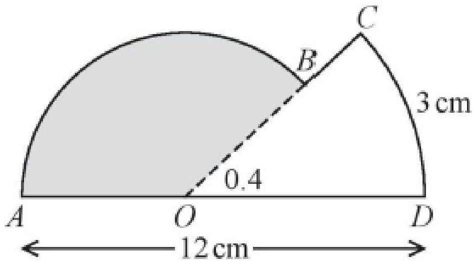
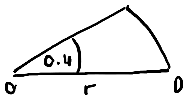


Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, $\angle COD$ = 0.4 radians and *AOD* is a straight line of length 12 cm, find

(*a*) the length of *OD*,



(2)

$S = r\theta$

$3 = r \times 0.4$

$\Rightarrow \ 3/0.4 = r \quad \Rightarrow \ r = \underline{7.5\text{cm}}$

(*b*) the area of the shaded sector *AOB*.

(3)

$AO = 12 - 7.5 = 4.5$ and Area of a sector is found using:

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(4.5)^2(\pi - 0.4) = 27.\underline{8\text{cm}^2}$

(Total for Question 1 is 5 marks)

---

2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos \theta = 2 \sin \theta.$$

The attempts of two of the students are shown below.

Student A	Student B
$\cos \theta = 2 \sin \theta$	$\cos \theta = 2 \sin \theta$
$\tan \theta = 2$	$\cos^2 \theta = 4 \sin^2 \theta$
$\theta = 63.4°$	$1 - \sin^2 \theta = 4 \sin^2 \theta$
	$\sin^2 \theta = \frac{1}{5}$
	$\sin \theta = \pm \frac{1}{\sqrt{5}}$
	$\theta = \pm 26.6°$

(*a*) Identify an error made by student A.

We know that $\tan x = \frac{\sin x}{\cos x}$ and Student A has wrote that $\frac{\cos x}{\sin x} = \tan x = 2$, **(1)**
but it should be $\frac{\sin x}{\cos x} = \tan x = \frac{1}{2}$.

Student B gives $\theta = -26.6°$ as one of the answers to $\cos \theta = 2 \sin \theta$.

(*b*) (i) Explain why this answer is incorrect.

$\cos(-26.6) \neq 2\sin(-26.6)$ Since $0.8942 \neq -0.8955$.

(ii) Explain how this incorrect answer arose.

(2)

This mistake comes from the Squaring of both sides.

(Total for Question 2 is 3 marks)

3. (a) Given that θ is small and in radians, show that the equation

$$\cos\theta - \sin\frac{1}{2}\theta + 2\tan\theta = \frac{11}{10} \qquad \text{(I)}$$

can be written as $5\theta^2 - 15\theta + 1 \approx 0$.

We have that $\sin\theta \approx \tan(\theta) \approx \theta$ and $\cos\theta \approx 1 - \frac{1}{2}\theta^2$. **(3)**

Then from equation I, we have $1 - \frac{\theta^2}{2} - \frac{\theta}{2} + 2\theta = \frac{11}{10}$

$\Rightarrow 10 - 5\theta^2 - 5\theta + 20\theta = 11$

$\Rightarrow 5\theta^2 - 15\theta + 1 = 0$ by rearranging.

The solutions of the equation $5\theta^2 - 15\theta + 1 = 0$ are 0.068 and 2.932, correct to 3 decimal places.

(b) Comment on the validity of each of these values as approximate solutions to equation (I).

$\theta = 0.068 \Rightarrow$ this will be a good approximation since θ is small. **(1)**

$\theta = 2.932 \Rightarrow$ this will not be a good approximation since θ is not small and we know the approximation is only valid for small angles.

(Total for Question 3 is 4 marks)

4. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)°, \quad 0 \leq t < 24,$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 a.m. and it takes 2 hours to load its cargo. The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

The boat enters at 6.30am and 6.30am is 6.5 hours after midnight. **(1)**

Therefore $t = 6.5$ and $D = 5 + 2\sin(30 \times 6.5) = 4.482\text{m}$

$= \underline{4.48\text{m}}$

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

When $D = 3.8$, we will have $3.8 = 5 + 2\sin(30t)$. **(4)**

$\Rightarrow \sin(30t) = -0.6 \Rightarrow 30t = \arcsin(-0.6)$

$\Rightarrow t = 7.228, 10.771.$

$t \geq 8.5$ because the boat arrives at 6.30pm and takes 2 hours to load the cargo,

So it cannot leave before 8.30pm $\Rightarrow t = 10.771.$

$\Rightarrow 10 + 60(0.771) \Rightarrow$ leaving time is $\underline{10{:}46\text{am}}$. **(Total for Question 4 is 5 marks)**

---

5. (*a*) Solve, for $-180° \leq \theta \leq 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*] **(6)**

$5\sin 2\theta = 9\tan\theta$ Rearrange

$\Rightarrow 5\sin 2\theta - 9\tan\theta = 0$

$\Rightarrow \dfrac{-9\sin\theta + 5\cos\theta\sin 2\theta}{\cos\theta} = 0$ make everything in terms of sin and cos

$\Rightarrow -9\sin\theta + 5\cos\theta\sin 2\theta = 0$ Expand using $\sin 2\theta - 2\sin\theta\cos\theta$

$\Rightarrow -9\sin\theta + 10\cos^2\theta\sin\theta = 0$

$\Rightarrow 10\cos^2\theta\sin\theta = 9\sin\theta \Rightarrow 10\cos^2\theta = 9 \Rightarrow \cos\theta = \sqrt{\frac{9}{10}} \Rightarrow \theta = \arccos\left(\sqrt{\frac{9}{10}}\right)$

$= 180 - 18.43 = 161.6°$ => $\theta = \underline{-18.4°}$ and $\theta = \underline{161.6°}$

(*b*) Deduce the smallest positive solution to the equation

$$5 \sin (2x - 50°) = 9 \tan (x - 25°)$$

(2)

$\theta = x - 25 = -161.6$ => $x = -131.6$ which is not positive.

$\theta = -18.4$ => $x = 6.6$ which is the smallest possible value.

(Total for Question 5 is 8 marks)

6.

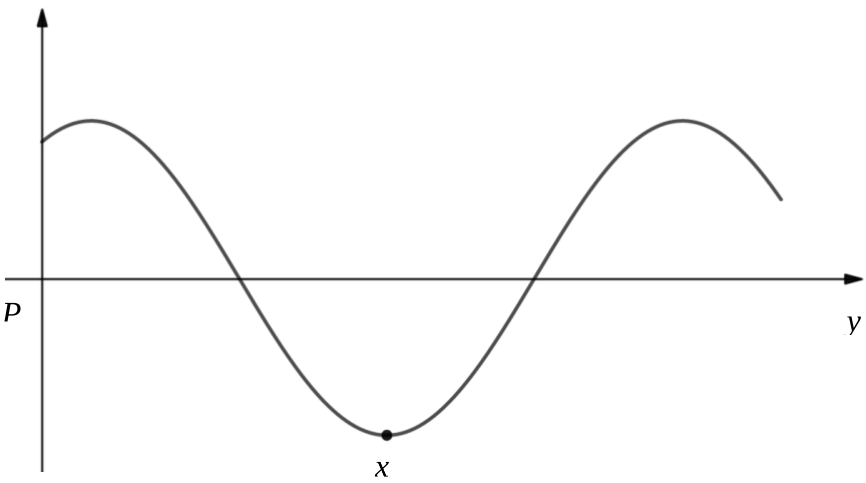


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 5\cos(x - 30)°, \qquad x \geq 0.$$

The point P lies on the curve and is the minimum point with smallest positive x-coordinate.

(a) State the coordinates of P.

(2)

The y-axis is stretched by a scale factor of 5 and the graph is shifted 30° to the right. => The usual $P = (180,-1)$ => $P = (180+30, -1 \times 5)$

$\Rightarrow P = \underline{(210, -5)}$

(b) Solve, for $0 \leq x < 360$, the equation

$$5\cos(x - 30)° = 4\sin x°,$$

giving your answers to one decimal place.

(4)

We use the compound angle formula => $5(\cos x \cos 30 + \sin x \sin 30) = 4\sin x$.

we expand the brackets and rearrange => $5\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) = 4\sin x$

$\Rightarrow \frac{5\sqrt{3}}{2}\cos x - \frac{3}{2}\sin x = 0$

$\Rightarrow \frac{5\sqrt{3}\cos x - 3\sin x}{2} = 0$

$\Rightarrow 5\sqrt{3}\cos x - 3\sin x = 0$

$\Rightarrow 5\sqrt{3}\cos x = 3\sin x$

$\Rightarrow \tan x = \frac{5\sqrt{3}}{3}$

$\tan x = \frac{\cos x}{\sin x}$ and use the inverse.

$\Rightarrow x = \arctan\left(\frac{5\sqrt{3}}{3}\right)$

$\Rightarrow x = \underline{70.9°}$ and $x = \underline{250.9°}$

(c) Deduce, giving reasons for your answer, the **number of roots** of the equation

$$5 \cos (2x - 30)° = 4 \sin 2x°$$

for $0 \leq x < 3600$.

(2)

Since 2x is inside the bracket the graph has been stretched by a factor of 1/2 in the x-axis. We normally would have 2 roots in $0 \leq x \leq 360$, and now 4 roots with the new transformation, then we multiply by 10 for $0 \leq x \leq 3600$.

=> 40 Roots

(Total for Question 6 is 8 marks)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)^\circ \qquad n \in \mathbb{Z}$$

$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \frac{\sin\theta\sin\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$ **(3)**

Noting that • $\operatorname{Cosec}\theta = \frac{1}{\sin\theta}$ $\qquad = \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

• $\operatorname{Cot}\theta = \frac{\cos\theta}{\sin\theta}$ $\qquad = \cos\theta \cdot \underline{\underline{\operatorname{Cot}\theta}}$ as required

• $\sin^2\theta + \cos^2\theta = 1$

(*b*) Hence, or otherwise, solve for $0 < x < 180°$

$$\operatorname{cosec} x - \sin x = \cos x \cot (3x - 50°)$$

(5)

We know $\operatorname{cosec}(x) - \sin(x) = \cos(x)\cot(x)$

So we can say that $\cos x \cot x = \cos x \cot(3x - 50°)$

$\Rightarrow \cot(x) = \cot(3x - 50)$

$\Rightarrow x = 3x - 50$

$\Rightarrow 2x = 50 \Rightarrow x = \underline{\underline{25°}}$

(Total for Question 7 is 8 marks)

8.

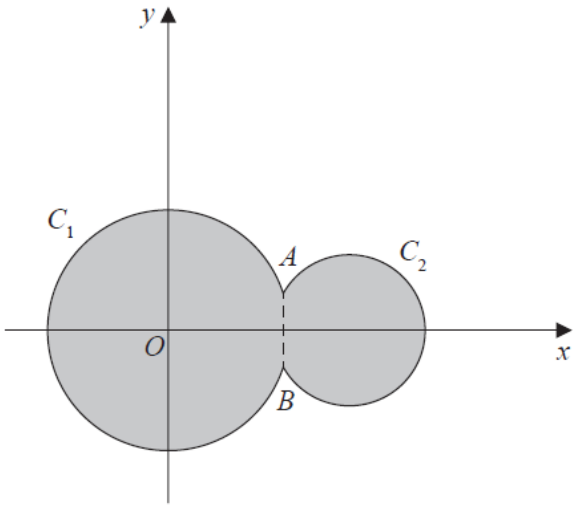


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

$y^2 = 100 - x^2$ and $y^2 = 40 - (x-15)^2$

$\Rightarrow 100 - x^2 = 40 - (x-15)^2$

$\Rightarrow 60 - x^2 = -x^2 + 30x - 225 \Rightarrow 30x = 285 \Rightarrow x = 9.5$

$\Rightarrow y^2 = 100 - (9.5)^2$

$\Rightarrow y = \pm\sqrt{9.75}$

Then $\bar{\theta} = \tan^{-1}\left(\frac{\sqrt{9.75}}{9.5}\right) \Rightarrow \bar{\theta} = 0.3176$.

Then $0.5\theta = \bar{\theta} = 0.3176 \Rightarrow \theta = \underline{0.635}$ radians

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

We know that angle AOB = 0.635, so we have that $(2\pi - 0.635) \cdot r$ is equal to the circumference of C1 $\Rightarrow 56.48$.

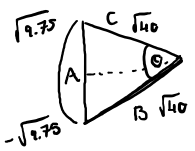
Then using coordinates we can create a triangle to work out an angle.

From the cosine rule : $\cos\theta = \frac{B^2 + C^2 - A^2}{2BC} \Rightarrow A = 2\sqrt{9.75}$, $B = C = \sqrt{40}$

and $\theta = 0.378$ radians.

(Total for Question 8 is 8 marks)

Then $2\pi - 0.378 = 5.89 \Rightarrow R \times 5.89 = 37.2$



$\Rightarrow$ Circumference $= 37.2 + 56.48 = \underline{93.7}$ units.

9. (i) Solve, for $0 \leq x < \frac{\pi}{2}$, the equation

$$4 \sin x = \sec x.$$

(4)

$$4\sin x - \sec x = 0$$

$$\Rightarrow \frac{-1 + 4\cos x \sin x}{\cos x} = 0$$

$$\Rightarrow 4\cos x \sin x - 1 = 0$$

$$\Rightarrow 2\sin 2x - 1 = 0$$

$$\Rightarrow \sin 2x = 1/2$$

$$\Rightarrow x = \frac{\arcsin(1/2)}{2} = \underline{\frac{\pi}{12}}$$

We make everything in terms of Cosx, then because the equation is equal to 0, we can eliminate the denominator Cosx.

We then use the double angle formula ($\sin 2x = 2\sin x \cos x$) and we finally rearrange and solve.

(ii) Solve, for $0 \leq \theta < 360°$, the equation

$$5 \sin \theta - 5 \cos \theta = 2,$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

$$5\sin\theta - 5\cos\theta = 2$$

Rearrange

$$\Rightarrow 5\sin\theta = 5\cos\theta + 2$$

Square and expand

$$\Rightarrow 25\sin^2\theta = (5\cos\theta + 2)^2$$

$$\Rightarrow 25\sin^2\theta - 4 - 20\cos\theta - 25\cos^2\theta = 0$$

$\sin^2\theta = 1 - \cos^2\theta$

$$\Rightarrow 21 - 20\cos\theta - 50\cos^2\theta = 0$$

$$\Rightarrow \cos\theta = \frac{-2 \pm \sqrt{46}}{10}$$

$$\Rightarrow \theta = \arccos\left(\frac{-2 \pm \sqrt{46}}{10}\right) \Rightarrow \theta = \underline{61.4°},\ \theta = \underline{151.4°},\ \theta = \underline{241.4°} \text{ and } \theta = \underline{331.4°}$$

(Total for Question 9 is 9 marks)

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

$\cos(3A) = \cos(2A + A) = \cos 2A \cos A - \sin 2A \sin A$

$= (2\cos^2 A - 1)\cos A - 2(\cos A \sin A)\sin A$

$= 2\cos^3 A - \cos A - 2\sin^2 A \cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A \equiv \underline{\underline{4\cos^3 A - 3\cos A}}$ as required.

(*b*) Hence solve, for $-90° \leq x \leq 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

From part a we have that the above is equivelant to

$1 - 4\cos^3 x - 3\cos x - (1 - \cos^2 x) = 0$

$\Rightarrow -4\cos^3 x + \cos^2 x + 3\cos x = 0.$

$\Rightarrow \cos x = 1, 0, -0.75 \Rightarrow x = \arccos(1, 0, -0.75)$

We solve this using substitution and inverse trig.

$\Rightarrow$ • $\cos(x) = 0 \Rightarrow x = \underline{\underline{-90°}}$ and $x = \underline{\underline{90°}}$.

• $\cos(x) = 1 \Rightarrow x = \underline{\underline{0°}}$

• $\cos(x) = -0.75 \Rightarrow x = \underline{\underline{138.6°}}$

(Total for Question 10 is 8 marks)

11. (*a*) Express 10 cos θ – 3 sin θ in the form R cos (θ + α), where $R > 0$ and $0 < \alpha < 90°$.
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

$10\cos\theta - 3\sin\theta$ and $\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad R = \sqrt{10^2+3^2} = \sqrt{109}$

$\Rightarrow 10 = R\cos\alpha$ and $3 = R\sin\alpha$

$\Rightarrow \frac{10}{\cos\alpha} = R$ and $\frac{3}{\sin\alpha} = R \Rightarrow \frac{10}{\cos\alpha} = \frac{3}{\sin\alpha} \Rightarrow \tan\alpha = \frac{3}{10}$

$\Rightarrow \alpha = 16.69° \Rightarrow \sqrt{109}\cos(\theta + 16.69)$

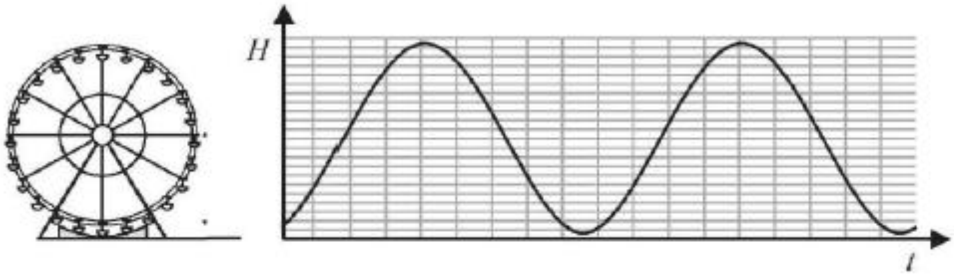


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(*b*) (i) find a complete equation for the model.

$H = a - \sqrt{109}\cos(80t + 16.69)$

Then when $t = 0$, $H = 1 \Rightarrow 1 = a - \sqrt{109}\cos(16.69) \Rightarrow a = 11.0$

$\Rightarrow \quad H = 11 - \sqrt{109}\cos(80t + 16.69)$

(ii) Hence find the maximum height of the passenger above the ground.

(2)

We want to maximise $H \Rightarrow$ we should minimise $\cos(80t + 16.69)$.
Its minimum will be -1 since it's a cos function

$H = 11 - \sqrt{109} \times -1 = 11 + \sqrt{109} = 21.4$ units

(*c*) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Max Height t Value :

$\cos(80t + 16.69) = -1 \implies 80t + 16.69 = 540$ (180 on 1st cycle). ($2 \times 360 - 180$)

$\implies$ 6.54 seconds which is equivalant to (3)

6 minutes and 32 seconds

It is decided that, to increase profits, the speed of the wheel is to be increased.

(*d*) How would you adapt the equation of the model to reflect this increase in speed?

You could increase speed by increasing the coefficient of t, i.e anything (1)

greater than the current 80 would work.

(Total for Question 11 is 9 marks)